

# Solar Nebula Magnetohydrodynamic Dynamos: Kinematic Theory, Dynamical Constraints, and Magnetic Transport of Angular Momentum

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A hydromagnetic dynamo provides the best mechanism for contemporaneously producing magnetic fields in a turbulent solar nebula. We investigate the solar nebula in the framework of a steady-state accretion disk model and establish the criteria for a viable nebular dynamo. We have found that typically a magnetic gap exists in the nebula, the region where the degree of ionization is too small for the magnetic field to couple to the gas. The location and width of this gap depend on the particular model; the supposition is that gaps cover different parts of the nebula at different evolutionary stages. We have found, from several dynamical constraints, that the generated magnetic field is likely to saturate at a strength equal to equipartition with the kinetic energy of turbulence. Maxwell stress arising from a large-scale magnetic field may significantly influence nebular structure, and Maxwell stress due to small-scale fields can actually dominate other stresses in the inner parts of the nebula. We also argue that the bulk of nebular gas, within the scale height from the midplane, is stable against Balbus-Hawley instability. © 1993 Academic Press, Inc.

## I. INTRODUCTION

It is widely believed that a primordial solar nebula, the precursor of the Sun and its planetary system, could be best described in terms of an accretion disk. According

to recent concepts, formation of single stars of about  $1 M_{\odot}$  results naturally in the formation of an accretion disk around them, which may then evolve to form a planetary system. Thus by studying circumstellar disks around solar-type pre-main-sequence stars we can deduce the basic physical properties that governed the dynamical state and behavior of the solar nebula. Astronomical observations (see, for example, review by Strom *et al.* 1989) strongly suggest that circumstellar disks around pre-main-sequence stars are in fact Keplerian accretion disks with sizes of the order of 100 AU, masses  $\sim 0.01$  to  $0.1 M_{\odot}$ , and evolutionary timescales about  $10^6$ – $10^7$  years. The disappearance of disks (and presumably the solar nebula) after such a relatively short period of time, if attributed to accretion processes, presents us with the problem of efficient, outward angular momentum transfer. Possible mechanisms acting to transport angular momentum include turbulent viscosity, gravitational torques, and magnetic torques.

In this paper we examine the problem of the existence and regeneration of magnetic fields in the solar nebula as described in the framework of an accretion disk model. The major motivation of our work is that by presenting some plausible criteria for the existence and character of nebular magnetic fields, we can start to address the problem of angular momentum transport via magnetic torques. Magnetic torques may not dominate the solar nebula dynamics during its formation stage, when the nebular disk is built up from infalling matter and its mass is comparable

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to the mass of the emerging protosun. Such a relatively massive disk is prone to large-scale gravitational instabilities that would globally and very efficiently redistribute angular momentum (Adams *et al.* 1989). However, as the external supply of matter is depleted and the outward angular momentum and inward mass transport caused by gravitational instabilities decrease the disk's mass below about 30% of the protosun mass, the disk becomes stable to all gravitational disturbances (Shu *et al.* 1990) and enters the so-called viscous stage. In this stage further evolution of the nebula has to be governed by either turbulent viscosity or magnetic fields.

It is usually assumed that accretion disks are turbulent despite the lack of rigorous proof that turbulence may, in fact, occur and without understanding what the source of the turbulence is. The most obvious candidate is differential rotation. It has been largely disregarded as a possible source of turbulence because the Keplerian rotation shear is stable with respect to *linear*, infinitesimal perturbation. However, it may be unstable with respect to *non-linear*, finite amplitude perturbations. Another candidate is convection driven by a superadiabatic temperature gradient across the disk (Ruden *et al.* 1988). Again, it is uncertain whether such a gradient can be maintained throughout a significant portion of the nebula.

Magnetic fields enter the question of nebula evolution in two rather distinct contexts. First, as originally postulated by Shakura and Sunyaev (1973), turbulent viscosity may originate from unspecified magnetic instabilities in partially ionized disks. This idea has been recently reinforced by Balbus and Hawley (1991) who observed that a local stability analysis of differentially rotating disks suggests the presence of shear instability (hereafter referred to as BH instability) whenever a weak vertical magnetic field is present. If present in astrophysical disks (for the argument that BH instability may be absent whenever an azimuthal magnetic field is present, see Knoblock 1992), this instability may pinpoint the source of the disk's turbulence in well-ionized disks. However, in conditions characteristic for the solar nebula, the BH instability may not operate (see section V). Second, as first pointed out by Levy (1978), magnetic fields maintained by nebular magnetohydrodynamic (MHD) dynamo can redistribute angular momentum by means of magnetic torque. It is the concept of a nebular dynamo that we investigate here. Because most parts of the nebula are rather weakly ionized, the question of whether MHD dynamo processes can operate there is nontrivial. Hayashi (1981) studied the ionization of nebular gas by cosmic rays alone and found that the magnetic field is coupled to the gas only in the outer parts of the nebula. Levy *et al.* (1991) discussed coupling of magnetic field and gas and dynamo action in the solar nebula; they concluded that while magnetic field would have been coupled to the gas everywhere in the

surface layers of the nebular disk—and that a dynamo magnetic field could be generated in the surface layers alone—the field is decoupled in those equatorial regions where the temperature is too low for significant thermal ionization and the surface density too high for cosmic rays to penetrate all the way to the midplane without great attenuation. Stepinski (1992a, hereafter referred to as Paper I) has addressed this question in the framework of a minimum mass, quiescent nebula model, and finds that for such a model the nebular dynamo can maintain a magnetic field only in the region located outside approximately 5 AU. He also calculated the regions of dynamo magnetic field amplifications for one specific accretion disk model of the solar nebula and found that they are located inside 1 AU and outside 5 AU, indicating that a large-scale magnetic field could be absent from the all-important region of the nebula between say 1 and 5 AU. To see how robust this result is we calculated criteria for dynamo magnetic field regeneration for a very broad range of nebular accretion disks. Our adopted nebula models form a family of geometrically thin, steady-state, turbulent accretion disks controlled by two major parameters:  $\alpha_{ss}$ , dimensionless measure of the strength of turbulent viscosity, and  $\dot{M}$ , a constant inward mass flux referred to hereafter as an accretion rate. Thus our models are based on the widely adopted  $\alpha$ -prescription of turbulent viscosity introduced by Shakura and Sunyaev (1973). We have decided to denote the dimensionless strength of turbulent viscosity by  $\alpha_{ss}$  (after Shakura and Sunyaev), and have reserved the symbol  $\alpha$  for the measure of the “so-called”  $\alpha$  effect widely used in dynamo theories (see section III).

The solar nebula was not static, but evolved with time; therefore, it is necessary to either consider evolutionary, time-dependent solutions for nebula structure, or calculate a large number of steady-state models and assume that at any given time the nebula can be described by one of those models. We opted for this second approach because it is mathematically simpler. We present our results on two-dimensional  $\alpha_{ss} - \dot{M}$  diagrams where any specific solar nebula model could be referred to by specifying a point  $(\alpha_{ss}, \dot{M})$  on a diagram.

We show in section IV that magnetic stress contributes very significantly to the angular momentum transport in the nebula, providing of course that nebular conditions are able to support a magnetic field. On the other hand, we have to appeal to some kind of turbulence to close the dynamo cycle (see section III) and make such support possible. The fact that we have to rely on turbulence of unknown origin to sustain the nebular dynamo may be viewed as a disappointment; one would prefer to have a physical process that would maintain the magnetic field independently from the postulated turbulence. However, such a process has not yet been found. There have been

attempts to close the dynamo cycle without turbulence. Vishniac *et al.* (1990) have pointed out that internal non-symmetric waves propagating inward within a disk could replace turbulence to achieve dynamo closure. However, such a mechanism requires the existence of a perturber located at the outer region of the disk. It is not clear what physical mechanism would provide the required perturbation in the solar nebula or any disk around a single star. Recently, Tout and Pringle (1992) have proposed a disk dynamo, in which they invoke the BH and Parker instabilities instead of turbulence to drive a dynamo. Again, this may be irrelevant for the solar nebula because, as we argue in section V, BH instability may not operate in the nebula. In addition, the dynamo scheme proposed by Tout and Pringle is basically a *postulated* process, whereas turbulent dynamos are relatively well-understood mechanisms that have been successfully applied to explain the magnetic fields of the Sun, Earth, and galaxies. Thus, we believe that, at present, turbulent dynamo formalism provides the best approach to the study of the existence of nebular magnetic fields and their role in angular momentum transport.

The main purpose of this paper is to establish criteria for a viable nebular dynamo. First, in section II, we present a description of accretion disks used in our study. Afterward, in section III, we apply dynamo formalism to identify regions of the nebula where a magnetic field can be sustained. In section IV we discuss the saturation mechanisms and estimate the magnitude of saturated magnetic fields. We then estimate the efficiency of magnetic angular momentum transport and compare it to viscous transport. We summarize our work in section V and discuss the physical significance of our results.

## II. SOLAR NEBULA AS AN ACCRETION DISK

Astronomical evidence and theoretical results suggest that accretion disks are a natural consequence of the gravitational collapse of dense rotating protostellar cores from which stars form. Therefore, we postulate that the solar nebula was in fact such an accretion disk. For the purpose of our calculations we assume the nebula to be a Keplerian, axisymmetric, geometrically thin, steady-state, turbulent disk.

The issue of existence of turbulence has been discussed in section I. Here we only add that viscous, low-mass disks are expected to be axisymmetric. As long as we restrict ourselves to low-mass disks, the velocity field should be Keplerian. Molecular line interferometry provides evidence of Keplerian rotation for at least two disks: HL Tau (Sargent and Beckwith 1987) and T Tau (Weintraub *et al.* 1989). Keplerian rotation of the disk material can occur only if centrifugal forces are much stronger than pressure gradients, which in turn means that the disk

is efficiently cooled and consequently geometrically thin. Thus the conditions “Keplerian” and “thin” are equivalent and very likely to be satisfied during the viscous stage of the solar nebula.

A thin accretion disk evolves on time scales  $\sim t_{\text{visc}} \sim R_0^2/\nu_0$ , where  $R_0$  is the disk’s radial length scale and  $\nu_0$  is the typical value of viscosity. If external conditions change on time scales longer than  $t_{\text{visc}}$ , the disk will settle into a steady-state structure. This is unlikely to happen for the solar nebula. If we identify  $t_{\text{visc}}$  with the survival time of circumstellar disks ( $10^6$ – $10^7$  years), external conditions (most notably mass supply rate) are changing faster, and a steady-state approximation is, in principle, invalid. Nevertheless, we have chosen a steady-state nebula model for its mathematical simplicity. To offset the apparent inapplicability of the steady-state assumption we calculate a very large number of different steady-state models. The philosophy here is that the evolution of the nebula can be approximated by the sequence of many stationary states. In addition, we do not attempt here to find a self-consistent nebula model. In the future, the self-consistent evolutionary nebula model, which includes magnetic forces, must be calculated, but that requires an understanding of nebular magnetic fields—the topic of this paper.

To specify the structure of the steady-state disk we need to supply the opacity law and a viscosity prescription. For the viscosity  $\nu$  we take a standard  $\alpha$ -prescription  $\nu = \alpha_{\text{ss}} C_s h$ , where  $C_s$  is the sound speed and  $h$  is the disk’s half-thickness. For the Rosseland mean opacity we have adopted the analytical piecewise-continuous power-law formulas given by Ruden and Pollack (1991). The opacity passes through six different regimes in order of increasing temperature. For the description of those regimes and specific forms of opacity law within them, we refer the reader to Appendix A of the Ruden and Pollack paper. Note that the opacity law we have adopted here differs from the simpler opacity law adopted in Paper I after Wood and Morfill (1988). The new opacities are more accurate in the higher-temperature regime, therefore more appropriate in the broad survey of many nebula models, some of them potentially having relatively hot regions. Consequently, in the regions of the nebular disk characterized by high temperatures, its structure, as described by our model, would differ from the structure prescribed by the Wood and Morfill model. It is important to note that the opacity law adopted here has been calculated using the fixed grain size. A more realistic model should take into account that coagulation and evaporation of grains may modify the opacity (Morfill 1988).

With our choice of opacity the structure of the nebula (midplane temperature  $T$ , half-thickness  $h$ , density  $\rho$ , and surface density  $\sigma_s$ ) can be found algebraically as functions of the radial distance from the protosun  $r$ ,

TABLE I  
Indices Describing the Solution of the Nebula Structure for  
Different Opacity Regimes

Quantity	C	$M$	$\dot{M}$	$\alpha_{ss}$	$r$
Regime I $r > r_1 = 7.68 M^{0.333} \dot{M}^{0.444} \alpha_{ss}^{-0.222}$					
$T$	3194.5	+0.500	+0.667	-0.333	-1.500
$h$	$1.97 \times 10^{12}$	-0.250	+0.333	-0.167	+0.750
$\rho$	$6.56 \times 10^{-12}$	+0.250	+0.000	-0.500	-0.750
$\sigma_s$	25.88	+0.000	+0.333	-0.667	+0.000
Regime II $r_1 > r > r_2 = 4.5 M^{0.333} \dot{M}^{0.444} \alpha_{ss}^{-0.222}$					
$T$	303.8	+0.115	+0.154	-0.077	-0.346
$h$	$6.08 \times 10^{11}$	-0.442	+0.077	-0.038	+1.327
$\rho$	$2.24 \times 10^{-10}$	+0.827	+0.769	-0.885	-2.481
$\sigma_s$	272.15	+0.385	+0.846	-0.923	-1.154
Regime III $r_2 > r > r_3 = 0.72 M^{0.333} \dot{M}^{0.451} \alpha_{ss}^{-0.216}$					
$T$	879.6	+0.353	+0.471	-0.235	-1.059
$h$	$1.035 \times 10^{12}$	-0.324	+0.235	-0.118	+0.971
$\rho$	$4.54 \times 10^{-11}$	+0.471	+0.294	-0.647	-1.412
$\sigma_s$	94.0	+0.147	+0.529	-0.765	-0.441
Regime IV $r_3 > r > r_4 = 0.134 M^{0.333} \dot{M}^{0.453} \alpha_{ss}^{-0.214}$					
$T$	1158.0	+0.080	+0.101	-0.058	-0.239
$h$	$1.19 \times 10^{12}$	-0.460	+0.050	-0.029	+1.381
$\rho$	$3.0 \times 10^{-11}$	+0.881	+0.849	-0.912	-2.642
$\sigma_s$	71.4	+0.420	+0.899	-0.942	-1.261
Regime V $r_4 > r > r_5 = 0.09 M^{0.333} \dot{M}^{0.476} \alpha_{ss}^{-0.191}$					
$T$	304.3	+0.300	+0.400	-0.200	-0.900
$h$	$6.09 \times 10^{11}$	-0.350	+0.200	-0.100	+1.050
$\rho$	$2.23 \times 10^{-10}$	+0.550	+0.400	-0.700	-1.650
$\sigma_s$	271.7	+0.200	+0.600	-0.800	-0.600
Regime VI $r < r_5$					
$T$	935.0	+0.144	+0.178	-0.111	-0.433
$h$	$1.07 \times 10^{12}$	-0.428	+0.089	-0.056	+1.283
$\rho$	$4.14 \times 10^{-11}$	+0.783	+0.733	-0.833	-2.35
$\sigma_s$	88.4	+0.356	+0.822	-0.889	-1.067

the mass of the protosun  $M$ , and parameters  $\dot{M}$  and  $\alpha_{ss}$ . The derivation of this algebraic solution is somewhat tedious; however, the results can be presented in relatively compact fashion. All physical quantities describing the structure of the nebula have the form

$$x = C M^l \dot{M}^k \alpha_{ss}^m r^n, \quad (1)$$

where  $x$  represents any physical quantity ( $T$ ,  $h$ ,  $\rho$ ,  $\sigma_s$ , etc.),  $C$  is a constant, and  $l$ ,  $k$ ,  $m$ , and  $n$  are power-law, real indices. The mass  $M$  is measured in  $M_\odot$ , accretion rate  $\dot{M}$  is measured in  $10^{-6} M_\odot/\text{year}$ , and radial distance  $r$  is given in astronomical units. Indices  $l$ ,  $k$ ,  $m$ , and  $n$  and the constant  $C$  change from one quantity to the other, as well as between different opacity regimes. The full set of their values for  $T$ ,  $h$ ,  $\rho$ , and  $\sigma_s$  is given in Table I.

We consider two different central masses, the first with  $M = 1$ , and the second with  $M = 0.8$ , recognizing that the protosun was still building up its mass through disk accretion. We examine accretion rates in the range from  $\dot{M} = 0.01$  to  $\dot{M} = 10$  (fiducial value being of the order of 1) and dimensionless viscosity in the range from  $\alpha_{ss} = 0.001$  to  $\alpha_{ss} = 1$  (fiducial value being of the order of 0.01 and the theoretical upper limit being equal to 1).

### III. CRITERIA FOR MAGNETIC FIELD REGENERATION

The relative strength of magnetic field “frozenness” into the fluid to its dissipation due to finite electrical conductivity of the fluid is measured by a magnetic Reynolds number  $\mathcal{R}_m = V_0 L_0 / \eta_0$ , where  $V_0$ ,  $L_0$ , and  $\eta_0$  are characteristic velocity, spatial scale, and magnetic diffusivity, respectively. The magnitude of  $\mathcal{R}_m$  in the solar nebula is in the range  $10$ – $10^3$ , *on average* about as high as in the Earth’s core, despite very low ionization levels. This is due to the large characteristic length scale (of the order of the disk’s half-thickness, or about 1 AU) and fast Keplerian rotation. Thus, nebular magnetic fields and nebular gas have a tendency to be coupled. The characteristic time over which such coupling persists is given by diffusion time  $t_{\text{diff}} = L_0^2 / \eta_0$ . For nebular conditions  $t_{\text{diff}} \approx 10$ – $10^3$  years, a very short time in comparison with nebular evolutionary timescales of about  $10^6$ – $10^7$  years. Therefore, it is difficult to see how any magnetic field originally contained in the nebular gas can persist long enough to produce significant dynamical effects, unless the magnetic field is contemporaneously regenerated by the dynamo mechanism.

The general method of treating accretion disk dynamos is given by Stepinski and Levy (1991; hereafter SL91). It was shown in SL91 that highly reduced (local) dynamo problem can be used to determine the region or regions of the disk where the magnetic field is able to be maintained. The reduced, one-dimensional dynamo equations depend parametrically on the radial coordinate  $r$ , and it is possible, using appropriate algebraic manipulations, to confine this parametric radial dependence to only one radially varying coefficient called the effective dynamo number  $D_{\text{eff}}$ ,

$$D_{\text{eff}} = \frac{3}{2} \frac{\alpha \omega h^3}{(\eta + \eta_{\text{turb}})^2}, \quad (2)$$

where the “so-called”  $\alpha$ -effect term,  $\alpha$ , describes the generation of a magnetic field due to helical turbulence,  $\eta$  and  $\eta_{\text{turb}}$  are resistive and turbulent magnetic diffusivities, and  $\omega$  is Keplerian angular velocity.

The effective dynamo number  $D_{\text{eff}}$  encompasses all the radial variation of the strength of regeneration mechanisms as compared to total diffusion. One of the results of SL91 was that, in the first approximation, a magnetic field can be maintained only in those parts of the nebula where  $D_{\text{eff}}$  exceeds a certain critical value,  $D_{\text{crit}}$ , which was calculated to be  $\sim 12$ . We use this result to extract a rather simple criterion for the existence of dynamo-generated fields in the solar nebula: for a given radial location in the nebula we can plot a contour  $D_{\text{eff}} = D_{\text{crit}}$  on the  $\alpha_{ss} - \dot{M}$  diagram, and all nebula models that find themselves encircled inside the contour would

permit the regeneration of a magnetic field, while those models located outside the contour would not. To accomplish this task we have to calculate  $D_{\text{eff}}$  as a function of  $r$ ; this involves determining the radial dependence of  $\alpha$  and  $\eta_{\text{turb}}$  from the model of turbulence and determining the radial dependence of  $\eta$  from the ionization state of the nebula.

The magnetic turbulent diffusivity  $\eta_{\text{turb}}$  is assumed to be identical to the general turbulent diffusion coefficient for a scalar field and equal to  $l_0 v_0$ , where  $l_0$  and  $v_0$  are turbulent mixing length and turbulent velocity, respectively. Shakura *et al.* (1978) suggested that, irrespective of the source of turbulence,  $v_0 \approx \omega l_0$ , so  $l_0/h \approx v_0/\omega h \approx v_0/C_s = M_t$ , where  $M_t$  is the turbulent Mach number, yielding  $\eta_{\text{turb}} = M_t^2 h^2 \omega$ . In the solar nebula turbulence is strongly affected by rotation (the dimensionless Rossby number is about unity). Under such conditions the radial dependence of the “ $\alpha$ -effect” is given by  $\alpha \approx l_0 v_0/h \approx M_t^2 h \omega$  (Ruzmaikin *et al.* 1988). In Paper I it was assumed that turbulent Mach number  $M_t$ , used in formulas for  $\eta_{\text{turb}}$  and  $\alpha$ , and dimensionless viscosity  $\alpha_{\text{ss}}$ , used to determine the nebula model, are independent. However, such an assumption is invalid because these same largest turbulent eddies are responsible for all turbulent viscosity, magnetic turbulent diffusivity, and creation of poloidal magnetic field. In fact, once we accept the Shakura *et al.* prescription for  $v_0$ ,  $\alpha_{\text{ss}} = M_t^2$ . Thus in our current calculations we use  $\alpha = \alpha_{\text{ss}} h \omega$  and  $\eta_{\text{turb}} = \alpha_{\text{ss}} h^2 \omega$ .

We consider cases of thermal and nonthermal ionization separately. This approach is justified because we show that every region in the nebula is dominated by only one of those processes. First we consider the criteria for magnetic field regeneration under the assumption that the coupling between the magnetic field and nebular gas is caused by thermal ionization. The temperature of the nebula is not high enough to cause thermal ionization of hydrogen, the main gas constituent. However, for nebular conditions, in regions where the temperature is about 1500 K, potassium becomes thermally ionized. At higher temperatures other alkali metals also become ionized. It turns out that the total ionization of potassium is adequate for providing magnetic coupling strong enough to maintain a viable dynamo. Thus, since our aim is to establish the minimum criteria for the onset of the dynamo process, we examine only the effect of the thermal ionization of potassium on the degree of ionization of the nebular disk. The addition of other alkali metals would not change those criteria. It is also clear that in the nebular regions that are able to maintain the total ionization of potassium, the contribution of nonthermal ionization sources to the nebular electrical conductivity is negligible. The degree of ionization of potassium  $x_p$  in the thermal equilibrium can be calculated from Saha’s equation

$$\log \left( \frac{x_p^2}{1 - x_p} \right) = -0.845 - \log \rho + \frac{3}{2} \log T - \frac{21878}{T}. \quad (3)$$

We assume that all potassium is in the gas phase, which is justifiable inasmuch as the thermal ionization is important only at high temperatures. The overall degree of ionization  $x = n_e/n_H$  is  $1.12 \times 10^{-7} x_p$ , where  $n_H$  is the hydrogen abundance,  $n_e$  is the electron abundance (equal to ion abundance), and  $1.12 \times 10^{-7}$  is the solar abundance of potassium relative to hydrogen.

For our sample of solar nebula models we calculate the degree of ionization using Eq. (3) and then calculate the electrical conductivity  $\sigma$  of nebular gas (for the relationship between  $x$  and  $\sigma$  see Paper I), and thus the magnetic diffusivity  $\eta = c^2(4\pi\sigma)^{-1}$ . Substituting  $\eta$  into Eq. (2) we obtain the effective dynamo number  $D_{\text{eff}}$  as a function of  $M$ ,  $\dot{M}$  and  $\alpha_{\text{ss}}$  and the radial coordinate  $r$ . The contours enclosing areas in the  $\alpha_{\text{ss}} - \dot{M}$  parameter space, where the effective dynamo number  $D_{\text{eff}}$  exceeds the critical value  $D_{\text{crit}}$ , can be seen in Fig. 1. Labeled contours connect all points  $\mathcal{P}$  (nebula models) for which  $D_{\text{eff}} = D_{\text{crit}}$  at the distance from the protosun as indicated by the label. For example, the contour labeled “3 AU” connects all nebula models such that at  $r = 3$  AU the effective dynamo number achieves its critical value and the magnetic field can be regenerated. All models represented by points inside a given contour are compatible with dynamo magnetic field regeneration at  $r$  as given by the contour label.

From Fig. 1 we conclude that the first obstacle to generating magnetic fields in the turbulent solar nebula by means of a MHD dynamo comes from the turbulence itself. Only models with  $\alpha_{\text{ss}} < 0.125$  are potentially capable of regenerating a magnetic field *somewhere* in the nebula. This is evident from the fact that no contours on Fig. 1 penetrate the region of the  $\alpha_{\text{ss}} - \dot{M}$  diagram for which the condition  $\alpha_{\text{ss}} > 0.125$  holds.

There is a simple explanation for the existence of such a limit. With a given strength of turbulence, the upper limit of  $D_{\text{eff}}$  is reached when the gas is highly conductive ( $\eta$  is negligible in comparison with  $\eta_{\text{turb}}$ ). The upper limit of  $D_{\text{eff}}$  is  $1.5\alpha_{\text{ss}}^{-1}$  regardless of the radial location (see eq (2)). Therefore, if  $\alpha_{\text{ss}} > 0.125$ , the upper limit of  $D_{\text{eff}}$  is smaller than  $D_{\text{crit}}$  and a magnetic field cannot be generated *anywhere* in the nebula. An additional obstacle to dynamo field generation in the nebula is the low electrical conductivity of the nebular gas, which, at present, is assumed to be due only to thermal ionization. Resulting high magnetic diffusivity further restricts nebula models capable of effective dynamo amplification. Figure 1 can be used to see whether a particular nebula model is able to support a dynamo cycle at a particular nebula location. In general, providing that  $\alpha_{\text{ss}} < 0.125$ , there exists the *maximum* regeneration radius, inside which electrical conductivity

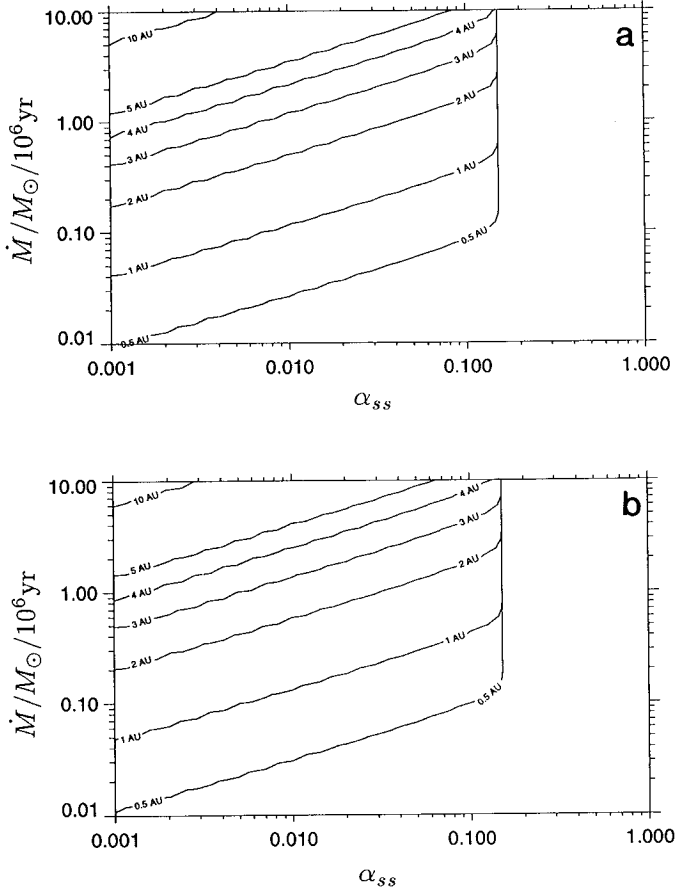


FIG. 1. Magnetic field regeneration regions due to thermal ionization on the  $\alpha_{ss} - \dot{M}$  diagram. Labeled contours connect all nebula models resulting in  $D_{\text{eff}} = D_{\text{crit}}$  at the distance  $r$  from the center as indicated by the label. All models represented by points inside a given contour are compatible with dynamo magnetic field regeneration at  $r$  as given by the label. (a) A nebula model with the central mass equal to  $1 M_{\odot}$  and (b) central mass equal to  $0.8 M_{\odot}$ .

is high enough and a magnetic field can be maintained by means of the dynamo mechanism. This maximum radius is largest for models characterized by high accretion rate and low  $\alpha_{ss}$ , and is smallest for models described by low accretion rate and high  $\alpha_{ss}$ . A nebula with  $\alpha_{ss} = 0.01$  and  $\dot{M} = 1$  can maintain a magnetic field within about 3 AU from the protosun. Beyond this radius a magnetic field cannot be maintained unless nonthermal ionization sources are able to provide the necessary conductivity.

We now consider the criteria for magnetic field regeneration assuming that the entire contribution to nebular electrical conductivity comes from nonthermal ionization sources: cosmic rays and the decay of radioactive nuclei such as  $^{26}\text{Al}$  and  $^{40}\text{K}$ . Formulas for the equilibrium degree of ionization  $x$  of nebular gas under such conditions, and subsequently the resulting magnetic diffusivity, have been given in Paper I. We calculate magnetic diffusivities for

our range of solar nebular models and substitute the results into Eq. (2) to obtain the effective dynamo numbers  $D_{\text{eff}}$  as a function of  $M$ ,  $\dot{M}$ ,  $\alpha_{ss}$ ,  $r$ , and the grain radius  $r_{\text{gr}}$ . Nebular grains enter the picture because one of two major free electron losses is due to recombination upon grain surfaces (see Paper I). This loss mechanism depends strongly on the size of the grains, being relatively large for small grains and relatively small for large grains. Grain size varies during the evolution of the nebula, from as little as  $5 \times 10^{-5}$  cm (the size of interstellar grains) to as much as 1 cm (the largest grain size still permitting ionization due to radioactive isotopes). We calculate criteria for magnetic field generation using four different values of  $r_{\text{gr}}$ : 1,  $10^{-2}$ ,  $10^{-3}$ , and  $5 \times 10^{-5}$  cm. (Note that we incorporate different grains sizes into the calculation of the equilibrium degree of ionization of nebular gas, but presently the opacity law uses one fixed grain radius).

Whereas thermal ionization of nebular gas extends more or less uniformly along the nebula thickness, nonthermal ionization is unlikely to be so uniform. In particular, the ionization of nebular gas due to cosmic rays decreases from the disk's surfaces to its midplane, while cosmic rays interact with nebular material and lose their energy. In our present model we ignore those effects. Our criteria are based on a model that calculates the degree of ionization at the nebula *midplane* and assumes that it extends *uniformly* along the entire disk thickness (from  $-h$  to  $+h$ ). Therefore, we underestimate the degree of ionization, and subsequently overestimate the value of the critical dynamo number  $D_{\text{crit}}$ . The value of 12 for  $D_{\text{crit}}$  is calculated under the assumption of uniform vertical distribution of degree of ionization. The dynamo model that would allow for ionization of nebular gas to increase away from the midplane would yield smaller  $D_{\text{crit}}$  (see Stepinski 1992b). Altogether, as long as we consider the bulk of the gas located within the scale height  $h$  from the midplane, those nonuniformities are not very large, and our approximation yields conservative, yet reasonable, regeneration criteria that are presented in Fig. 2 for central mass equal to  $1 M_{\odot}$ , and in Fig. 3 for central mass equal to  $0.8 M_{\odot}$ . Those figures consist of four  $\alpha_{ss} - \dot{M}$  diagrams for four different grains sizes: (a) 1 cm, (b)  $10^{-2}$  cm, (c)  $10^{-3}$  cm, and (d)  $5 \times 10^{-5}$  cm. Different line styles denote contours enclosing nebula models capable of maintaining magnetic fields at corresponding distances from the central protosun.

Once again, the strict limit of  $\alpha_{ss} < 0.125$  for the working dynamo can be observed on all panels presented in Figs. 2 and 3. However, contrary to criteria established for the case of thermal ionization (Fig. 1), now we have found that there exists a *minimum* radius beyond which a magnetic field can be maintained by a MHD dynamo.

The existence of such a minimum regeneration radius has the following simple explanation. Cosmic rays are the major ionization source, their effectiveness in ionizing the

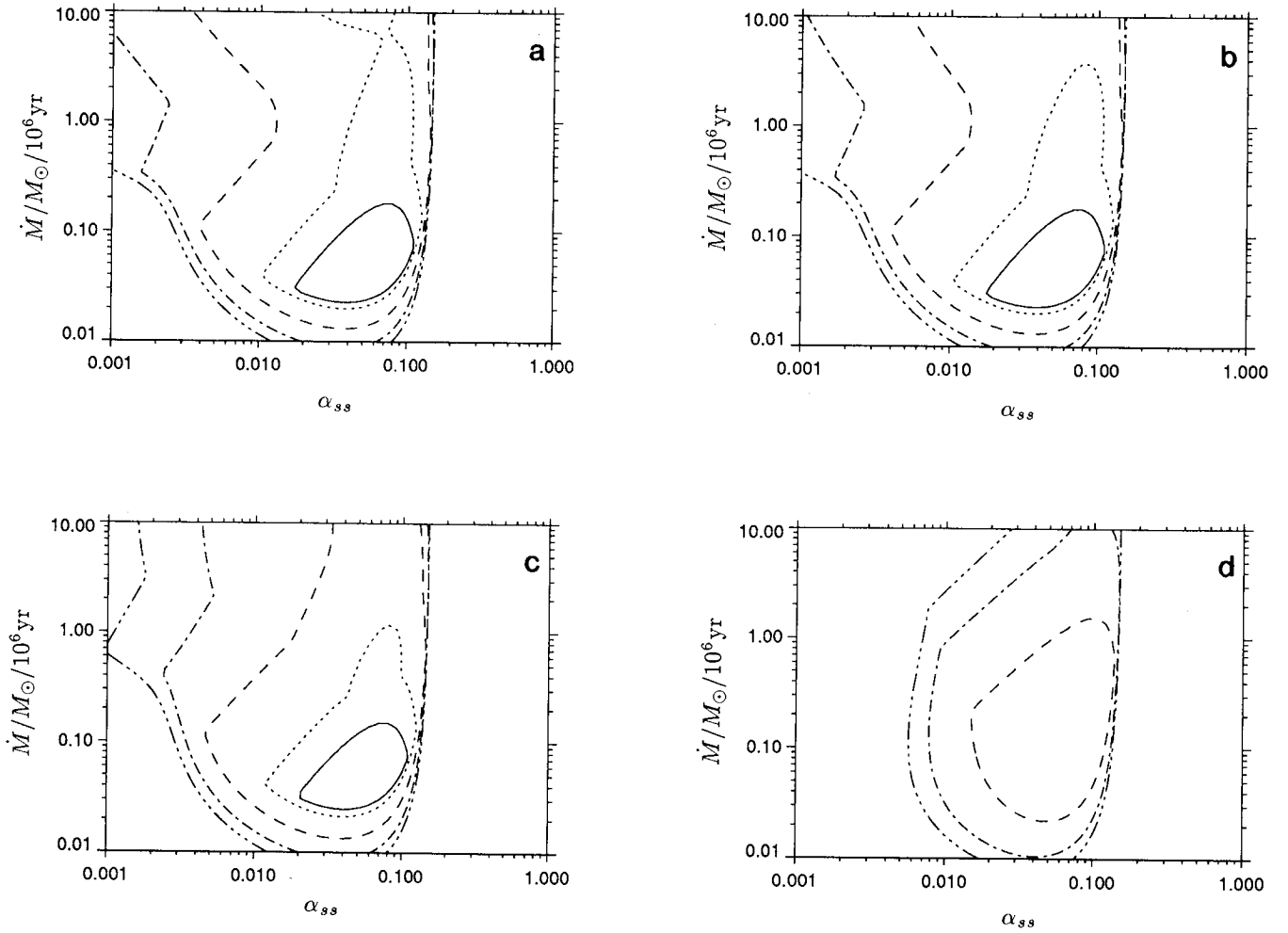


FIG. 2. Magnetic field regeneration regions due to nonthermal ionization on the  $\alpha_{ss} - \dot{M}$  diagram. (a–d) Calculations for four different grains sizes: (a) 1 cm, (b)  $10^{-2}$  cm, (c)  $10^{-3}$  cm, and (d)  $5 \times 10^{-5}$  cm. Different line styles connect all nebula models resulting in  $D_{\text{eff}} = D_{\text{crit}}$  at the different distances  $r$  from the center: long dash, 3 AU; solid line, 4 AU; dotted, 5 AU; short dash, 10 AU; dash-dot, 20 AU; dash-dot-dot-dot, 30 AU. All models represented by points inside a given contour are compatible with dynamo magnetic field regeneration at the given distance from the center. The mass of the protosun is assumed to be  $1 M_{\odot}$ .

midplane nebula region depends on the surface density  $\sigma_s$ , which is a decreasing function of  $r$  (except in Regime I where it is constant; see Table I). Thus, the outermost parts of the nebula provide very little shielding from cosmic rays, allowing relatively high ionization of the midplane regions. This shielding increases inward, decreasing the degree of ionization of the midplane gas. At a certain radius the ionization is just too small to support the dynamo. Additionally, unshielded ionization by radioactive elements is by itself inadequate to maintain a dynamo.

Let us first discuss the case of a  $1 M_{\odot}$  protosun (Fig. 2). For small grains (Fig. 2d) there are no nebula models in which a dynamo can maintain a magnetic field in the part of the nebula closer than 10 AU. There is a set of models that can support a dynamo operating at 10 AU, this set increases for nebular regions located farther from

the center. For larger grains, with radii  $10^{-3}$  cm or more (Figs. 2a–2c), the electron losses due to recombination upon grain surfaces decrease rapidly, leaving the ion–electron reaction as the dominant loss mechanism. This manifests itself in larger ionization levels, and consequently in smaller minimum regeneration radii. There is a small set of models with a minimum regeneration radius of about 4 AU. These models are characterized approximately by  $0.03 < \alpha_{ss} < 0.1$  and  $0.02 < \dot{M} < 0.1$ , and they describe the solar nebula with a mass of a few percent of  $M_{\odot}$ . Progressively larger sets of models permit magnetic field regeneration beyond correspondingly larger minimum radii. The fiducial nebula with  $\alpha_{ss} = 0.01$  and  $\dot{M} = 1$  can maintain magnetic fields beyond slightly more than 10 AU, provided that grains accumulated to sizes larger than  $10^{-3}$  cm.

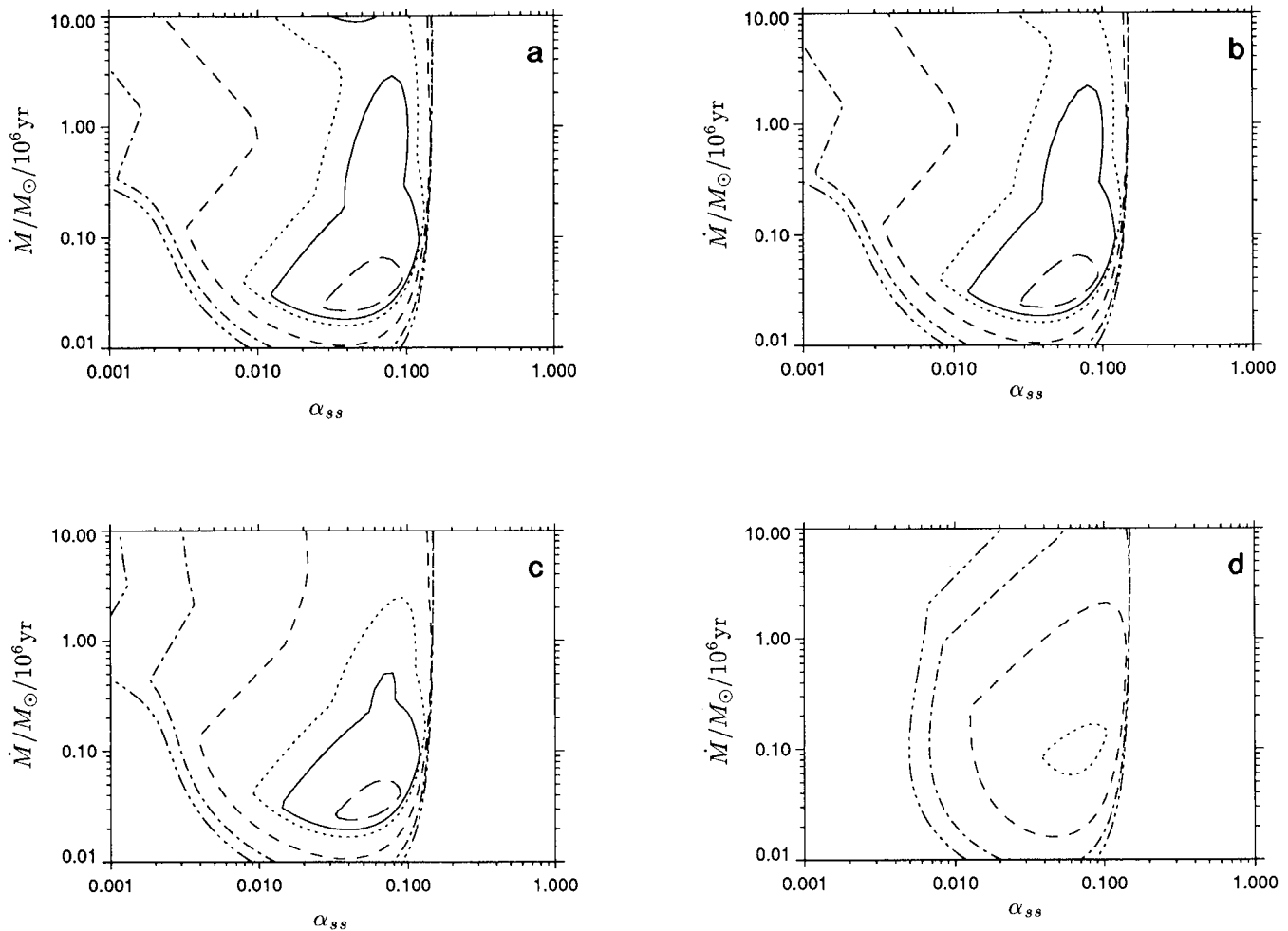


FIG. 3. Magnetic field regeneration regions due to nonthermal ionization on the  $\alpha_{ss} - \dot{M}$  diagram. Same as Fig. 2, but for the mass of the protosun equal to  $0.8 M_{\odot}$ .

The case of a  $0.8 M_{\odot}$  protosun (Fig. 3) is quantitatively very similar to the one discussed above. However, since the nebular surface density decreases with the decrease of  $M$  (see Table I), we expect all critical regeneration radii to be smaller. This is substantiated by Fig. 3. For the smallest grains considered (Fig. 3d), there are now models capable of sustaining magnetic field at 5 AU and beyond. For larger grain sizes (Figs. 3a–3c) models can be found that can maintain a magnetic field as close as 3 AU from the protosun. However, the critical regeneration radius for the fiducial nebula with  $\alpha_{ss} = 0.01$  and  $\dot{M} = 1$  is about 10 AU, only slightly smaller than for the case of  $1 M_{\odot}$  protosun.

Comparing Figs. 2 and 3 with Fig. 1 one can observe that nebular regions able to support a dynamo because of the sufficient thermal ionization and those able to support a dynamo due to sufficient nonthermal ionization are unconnected. For the overwhelming majority of models, the maximum regeneration radius related to thermal ionization is

smaller than the minimum regeneration radius related to nonthermal ionization. Consequently, nebular regions between maximum thermal radius and minimum nonthermal radius define the gap in which a magnetic field cannot be maintained. The width and location of those gaps vary from model to model. For example, the fiducial nebula has a gap between  $\sim 3$  and  $\sim 10$  AU. The models with the smallest minimum nonthermal regeneration radii have gaps between  $\sim 0.5$  and  $\sim 4$  AU. The models with the largest maximum thermal regeneration radii have gaps between  $\sim 10$  and  $\sim 20$  AU. Some models have no gap and are able to maintain a magnetic field throughout the entire radial extent of the nebula. They are shown in Fig. 4 by the shaded area on the  $\alpha_{ss} - \dot{M}$  diagram. This figure also shows the contours of constant nebular mass. In calculating nebular masses we have assumed the inner radius of the nebula to be equal to 0.02 AU and the outer radius to be equal to 30 AU. Generally, models without gaps correspond to high  $\dot{M}$ , and consequently have disk masses too high to be of any

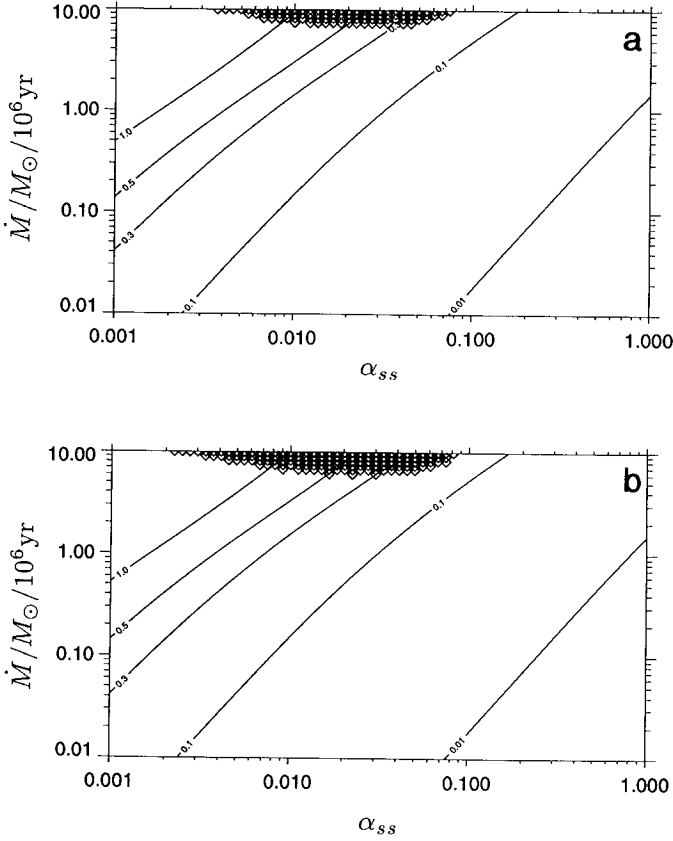


FIG. 4. Models of the solar nebula capable of maintaining magnetic field throughout the entire radial extent of the nebula are shown by the shaded area on the  $\alpha_{ss} - \dot{M}$  diagram. Labeled contours connect all nebula models resulting in the same nebular mass. (a) A nebula model with the central mass equal to  $1 M_\odot$  and (b) a central mass equal to  $0.8 M_\odot$ . Grain size equal to 1 cm is assumed.

interest to us. However, among those “gapless” models there are a few (those corresponding to  $\alpha_{ss}$  close to 0.1) that have nebular masses in the  $0.3 M_\odot$  range and thus can be relevant to the “viscous stage” discussed in this paper.

In summary, the present calculation found that the existence of a gap in the solar nebula, where magnetic fields cannot be maintained, is a robust feature of a nebular dynamo. Such a gap has been found for the overwhelming majority of models, regardless of assumed grain size or the mass of the central protosun. Very few models that are physically relevant to the viscous stage of the solar nebula have no gaps and are actually capable of maintaining a magnetic field throughout the entire nebula. However, we have also found that the width and location of a “no-magnetic-field” gap are very different for different models; therefore the findings of Paper I that magnetic fields are absent from the region of the nebula between  $\sim 1$  and  $\sim 5$  AU is not a robust result. Instead, we expect that as the solar nebula evolves, the gap in the dynamo regeneration region shifts from one location to the other.

We review qualitatively one hypothetical scenario of such evolution. Assume the nebula that has a fixed  $\alpha_{ss} = 0.06$  throughout its evolution. The particular value has been taken from the recent paper by Dubrulle (1992) and was obtained from the turbulent closure model. Assume further that the nebula starts its viscous stage with a high  $\dot{M}$  and accretion rate decreases during the evolution. Let us consider four models (points  $\mathcal{P}(\alpha_{ss}, \dot{M})$  on the  $\alpha_{ss} - \dot{M}$  diagram), chosen to correspond to nebular evolution. The initial point  $\mathcal{P}_1(0.06, 10)$  describes a nebula with a mass  $M_d$  (expressed in solar masses) equal to about 0.2. There is no magnetic gap; MHD dynamo can operate everywhere in this nebula. Next, the nebula evolves to point  $\mathcal{P}_2(0.06, 1)$ . This model gives a nebula with  $M_d \approx 0.06$ , and there is a magnetic gap between  $\sim 2$  and  $\sim 4$  AU. When the nebula evolves to point  $\mathcal{P}_3(0.06, 0.1)$ ,  $M_d$  decreases to about 0.03 and the gap changes to cover the region between  $\sim 0.6$  and  $\sim 3.5$  AU. Finally, the nebula arrives at point  $\mathcal{P}_4(0.06, 0.01)$ .  $M_d$  is about 0.01 and only the innermost parts of the nebula, up to about 0.2 AU, can support the magnetic dynamo. Nebula models with a very small  $\dot{M}$  cannot maintain a dynamo in the region where nonthermal ionization dominates (which is almost everywhere), not because of insufficient ionization, but because of the small vertical size  $h$  (see Eq. (2)).

#### IV. DYNAMICAL CONSTRAINTS AND MAGNETIC TRANSPORT OF ANGULAR MOMENTUM

Kinematic dynamo equations are linear and homogeneous; consequently, the amplitude of the generated magnetic field is not determined by this theory. According to kinematic theory, in the nebular regions where  $D_{\text{eff}} > D_{\text{crit}}$ , a dynamo will amplify a seed magnetic field without a limit. In reality, the Lorentz forces must eventually become great enough to modify amplification sources, reducing dynamo action and halting the growth of the magnetic field. In addition, in the presence of strong enough magnetic fields, some nonlinear magnetic field losses become important, saturating its further growth. We estimate the ultimate strength of the generated field by considering several physically relevant dynamical constraints on its growth.

##### Equipartition

Among several ideas how to estimate the induced magnetic field strength, the most widely accepted is the idea of “equipartition,” where the Alfvén velocity  $v_A = B/\sqrt{4\pi\rho}$  is assumed to be of the order of the turbulent velocity  $v_0$ . Note that according to this definition the critical magnetic field is in the equipartition with the fluctuating kinetic energy density  $\rho v_0^2$ , and not with the thermal pressure  $P$ . Mean magnetic fields of more or less equiparti-

tion values have been observed in our Galaxy, as well as in other galaxies, suggesting that galactic dynamos saturate when equipartition is achieved. This opens up the possibility that other disk dynamos, and in particular the nebular dynamo, may saturate at this limit. The turbulent velocity  $v_0$  equals  $M_t C_s$  or, according to our model of turbulence,  $\alpha_{ss}^{1/2} C_s$ . Thus the radial dependence of the equipartition magnetic field strength,  $B_{eq}$ , is given by

$$B_{eq} = \left( \frac{4\pi k}{\mu m_H} \right)^{1/2} \alpha_{ss}^{1/2} \rho^{1/2} T^{1/2}, \quad (4)$$

where  $\mu$  is the mean molecular weight,  $m_H$  is the mass of hydrogen atom, and  $k$  is Boltzmann's constant.

#### Coriolis versus Lorentz forces

Dynamo generation of large-scale magnetic fields depends on the organizing influence of the Coriolis force to produce the helical turbulence. The magnetic field, in turn, is a source of stress, which opposes the Coriolis force in the production of those helical turbulent eddies. The strength of saturated magnetic field,  $B_c$ , is given in order of magnitude by the balance of Coriolis and Lorentz forces acting on an eddy with size  $l_0$  and turbulent speed  $v_0$ ,

$$\frac{B_c^2}{4\pi h} = \rho v_0 \omega. \quad (5)$$

Using the relationships  $v_0 = \alpha_{ss}^{1/2} C_s$  and  $h\omega = C_s$  we find that  $B_c = \alpha_{ss}^{-1/4} B_{eq}$ . Thus, the dynamical limit derived from the balance of Coriolis and Lorentz forces has a radial dependence identical to the radial dependence calculated for the equipartition limit, but its magnitude is larger by the factor  $\alpha_{ss}^{-1/4}$ . Considering that  $\alpha_{ss}$  is not likely to be much smaller than 0.001,  $B_c$  and  $B_{eq}$  are within this same order to magnitude.

#### Magnetic versus Gas Pressures

The vertical structure of the nebula is strongly affected by the magnetic field if  $B^2/8\pi$  is comparable to or larger than the gas pressure. Let us estimate the value of the magnetic field  $B_{pr}$  for which magnetic pressure equals the gas pressure (equipartition with thermal pressure), assumed to be given by equation of state for the perfect gas

$$\frac{B_{pr}^2}{8\pi} = \frac{\rho k T}{\mu m_H}. \quad (6)$$

It is easy to show that  $B_{pr} = \sqrt{2} \alpha_{ss}^{-1/2} B_{eq}$  and  $B_{pr} = \sqrt{2} \alpha_{ss}^{-1/4} B_c$ . Therefore,  $B_{pr}$ ,  $B_c$ , and  $B_{eq}$  all have this same

radial dependence; however,  $B_{pr} > B_c > B_{eq}$ . For relatively large values of  $\alpha_{ss}$  all those limits are within this same order of magnitude; for values of  $\alpha_{ss}$  around 0.001 there is about an order of magnitude difference between  $B_{pr}$  and  $B_{eq}$ .

In solving the magnetic dynamo problem (Section III) we have considered two mechanisms of magnetic field losses: resistive and turbulent diffusivities. Those losses are described by diffusion coefficients that are independent of the magnetic field strength, and thus are dominant magnetic losses in a weak field regime. However, for relatively strong magnetic fields nonlinear loss mechanisms such as magnetic buoyancy and ambipolar diffusion become important. We estimate the strength of the magnetic field at which those nonlinear mechanisms become as important as the combined action of resistive and turbulent diffusion. These magnetic field values provide us with additional dynamical constraints on the overall equilibrium magnitude of the induced magnetic field.

#### Magnetic Buoyancy

The loss of magnetic flux due to magnetic buoyancy has been described by Parker (for a full description, see Parker 1979). In the presence of vertical pressure gradients, the density of nebular gas within a magnetic flux tube that is being stretched by differential rotation is lower than that of its surrounding. As a result, the tube is rising with approximately Alfvén velocity  $v_A$  and is expelled from the nebula on a timescale  $\tau_{buoy}$  of the order of  $\xi h/v_A$ , where  $\xi$  is a constant of the order of  $|B_\phi/B_r| \approx 1/M_t$  (for this last approximation, see Pudritz 1981b). We can define "buoyant diffusivity"  $\eta_{buoy} = h^2/\tau_{buoy}$ , and calculate the value of magnetic field  $B_{buoy}$  defined by the equality  $\eta_{buoy} = \eta + \eta_{turb}$ . Assuming once again  $M_t = \alpha_{ss}^{1/2}$  we have

$$B_{buoy} = \left( \frac{4\pi\rho}{\alpha_{ss}} \right)^{1/2} \frac{\eta + \eta_{turb}}{h}. \quad (7)$$

In the regions of the nebula where  $\eta_{turb} \gg \eta$ , Eq. (7) gives  $B_{buoy} = B_{eq}$ ; otherwise  $B_{buoy} > B_{eq}$ . If the magnetic field is allowed to grow to a magnitude larger than  $B_{buoy}$ , magnetic buoyancy removes magnetic flux faster than it can be regenerated (according to the linear dynamo), resulting in a saturation of magnetic field at a value of  $\sim B_{buoy}$ .

#### Ambipolar Diffusion

In a lightly ionized solar nebula magnetic fields are coupled directly only to the charged particles. Collisions of ions with the neutrals exert a frictional force on the neutrals, which effectively allows the Lorentz force ex-

erted by magnetic field to influence the bulk of the matter. There is, however, a steady slip of neutrals with respect to the magnetic field, an ambipolar diffusion, that is characterized by the drift velocity  $v_{\text{drift}} \approx B^2/4\pi\rho_i\chi_{\text{in}}h$  (see Spitzer 1978) given by the balance between Lorentz and collisional forces. The quantity  $\rho_i$  is the mass density of the charged component and is equal to  $x\rho$ . The ion-neutral collision rate,  $\chi_{\text{in}}$ , is equal to  $\langle\sigma v\rangle_{\text{in}}\rho/m_{\text{H}}$ , where  $\langle\sigma v\rangle_{\text{in}}$  is the mean of the product of the relative velocity and the collision cross section of ions with respect to neutrals. The loss of the magnetic field due to ambipolar diffusion can be described as the escape of magnetic flux from the nebula on a time scale  $\tau_{\text{amb}} = h/v_{\text{drift}}$ . We can define “ambipolar diffusivity”  $\eta_{\text{amb}} = h^2/\tau_{\text{amb}}$  and calculate the value of magnetic field  $B_{\text{amb}}$  defined by the equality  $\eta_{\text{amb}} = \eta + \eta_{\text{turb}}$ ,

$$B_{\text{amb}} = \left( \frac{4\pi\langle\sigma v\rangle_{\text{in}}}{m_{\text{H}}} \right)^{1/2} x^{1/2} \rho (\eta + \eta_{\text{turb}})^{1/2}. \quad (8)$$

In the regions of the nebula where  $\eta \gg \eta_{\text{turb}}$ , the value of  $B_{\text{amb}}$  is given by  $1.1 \times 10^{10} \rho$ , assuming that  $\langle\sigma v\rangle_{\text{in}} \approx 1.3 \times 10^{-9} \text{ cm}^3/\text{sec}$  (Hayashi 1981). In the regions of the nebula where  $\eta_{\text{turb}} > \eta$ , the value of  $B_{\text{amb}}$  is larger. For magnetic fields larger than  $B_{\text{amb}}$ , the drift between neutral bulk of the gas and magnetic field is significant, and the coupling between neutrals, ions, and magnetic fields fails, and the dynamo process ceases. Therefore,  $B_{\text{amb}}$  provides yet another dynamical constraint on the overall magnitude of the generated field.

Figure 5 illustrates the radial dependence of  $B_{\text{c}}$ ,  $B_{\text{buoy}}$ , and  $B_{\text{amb}}$  for four nebula models described in section III. These are the models with  $\alpha_{\text{ss}} = 0.06$  and different accretion rates. We excluded  $B_{\text{eq}}$  and  $B_{\text{pr}}$  from Fig. 5 because they follow the radial dependence of  $B_{\text{c}}$  with only a small vertical shift. Because dynamical constraints, as described above, are obtained from considerations that are *independent* from any dynamo criteria, we calculate them everywhere in the nebula, but they apply only in the regions where magnetic field can be maintained according to the criteria described in section III. It is interesting to observe that, in the regions where magnetic field is generated, dynamical constraints show that the saturation strength of magnetic field should be of the order of the equipartition field  $B_{\text{eq}}$ .

We now consider whether dynamo-generated magnetic fields could have substantial effects on the structure and dynamical evolution of the solar nebula. The dynamical evolution of the nebula is governed by the transport of angular momentum, which in turn is associated predominantly with the  $\phi r$ -component of the stress tensor. In a standard accretion disk theory, like the one used in Section II of this paper, angular momentum is transported

by means of turbulent viscous stress tensor. The  $\phi r$ -component of this tensor is  $\mathcal{T}_{\nu} = \omega\rho\nu$ . Kinematic turbulent viscosity  $\nu$  is numerically equal to turbulent magnetic diffusivity  $\eta_{\text{turb}}$ , so  $\mathcal{T}_{\nu} = \omega\rho\eta_{\text{turb}} = \alpha_{\text{ss}}\rho C_s^2$ . We estimate the relative importance of turbulent viscous stress tensor and the Maxwell stress tensor due to magnetic fields permeating the nebula.

First let us consider the  $\phi r$ -component of the Maxwell stress,  $\mathcal{T}_{\text{B}} = B_{\phi}B_r/4\pi$ , due to large-scale, mean fields generated by a nebular dynamo. In the  $\alpha\omega$ -type dynamo, the  $\phi$ -component of the magnetic field dominates, so we can take  $B_{\phi} \approx B$ , where  $B$  is the total strength of the large-scale magnetic field. The radial component of magnetic field  $B_r$  is of the order of  $M_{\text{t}}B_{\phi}$  (Pudritz 1981b), so  $\mathcal{T}_{\text{B}} \approx \alpha_{\text{ss}}^{1/2}B^2/4\pi$ . The ratio of Maxwell stress calculated for the large-scale magnetic field to turbulent viscous stress is

$$\frac{\mathcal{T}_{\text{B}}}{\mathcal{T}_{\nu}} = \alpha_{\text{ss}}^{1/2} \left( \frac{B}{B_{\text{eq}}} \right)^2. \quad (9)$$

If the generated magnetic field saturates at the equipartition strength, this ratio is equal to  $\alpha_{\text{ss}}^{1/2}$ . We cannot exclude the possibility that the saturation of magnetic field occurs at values larger than  $B_{\text{eq}}$ ; however, from the discussion of dynamical constraints, it is clear that it cannot be much larger. Thus, it seems that mean field stress would not *dominate* the viscous stress in the angular momentum transport in the nebula, but for some evolutionary stages it may play a role *comparable* to that played by turbulence.

Second, let us consider the Maxwell stress tensor due to the small-scale, fluctuating magnetic fields. In a turbulent nebula a large-scale field is inevitably accompanied by small-scale, random component of the magnetic field,  $b$ , with zero mean. The random magnetic field fluctuation having a mean of zero does not necessarily have a vanishing  $\langle b^2 \rangle$  or  $\langle b_{\phi}b_r \rangle$ . In fact, it was shown by Krause and Roberts (1976) that  $\langle b^2 \rangle \sim (\eta_{\text{turb}}/\eta)B^2$ , so when  $\eta_{\text{turb}} > \eta$ , large magnetic fluctuations occur. In the presence of strong magnetic fluctuations, the Maxwell stress due to them,  $\mathcal{T}_{\text{b}} = \langle b_{\phi}b_r \rangle/4\pi$ , has been estimated by Pudritz (1981a) as  $\mathcal{T}_{\text{b}} \sim \alpha_{\text{ss}}(1/4\pi)(h/r)^2 \langle b^2 \rangle$  and can exceed the Maxwell stress associated with the large-scale field. The ratio of Maxwell stress due to small-scale fields to turbulent viscous stress is

$$\frac{\mathcal{T}_{\text{b}}}{\mathcal{T}_{\nu}} = \alpha_{\text{ss}} \left( \frac{B}{B_{\text{eq}}} \right)^2 \left( \frac{h}{r} \right)^2 \left( \frac{\eta_{\text{turb}}}{\eta} \right). \quad (10)$$

Assuming large-scale fields of equipartition strengths, we calculated the radial dependence of  $\mathcal{T}_{\text{b}}/\mathcal{T}_{\nu}$  for nebula models with  $\alpha_{\text{ss}} = 0.06$  and accretion rate  $\dot{M}$  equal to 10, 1, 0.1, and 0.01, respectively. Those are the four

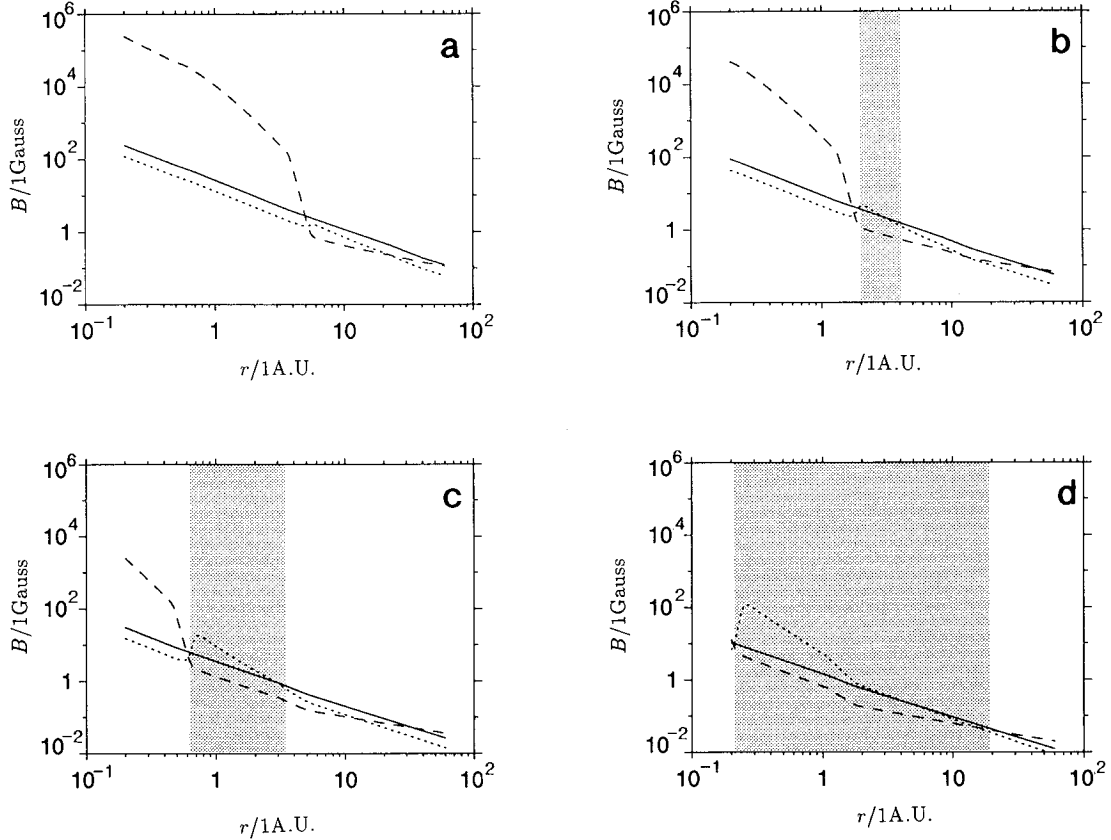


FIG. 5. The strength of magnetic fields in the solar nebula as a function of distance from the protosun derived from different dynamical constraints. The solid line corresponds to  $B_c$ , the dotted line corresponds to  $B_{\text{buoy}}$ , and the dashed line corresponds to  $B_{\text{amb}}$ . (a–d) Calculations for four nebula models with  $\alpha_{\text{ss}} = 0.06$  and  $M$  equal to 10, 1, 0.1, and 0.01, respectively. Grain size equal to 1 cm is assumed. The shaded areas represent regions of the nebula where magnetic field cannot be maintained.

nebula models corresponding to a hypothetical evolutionary scenario (see Section III). Results are presented in Fig. 6. We conclude that fluctuating Maxwell stress may dominate turbulent viscous stress in the region of the nebula where the dynamo operates because thermal ionization is high enough. In the outer nebula regions, which maintain a magnetic field due to nonthermal ionization, the degree of ionization is enough for the dynamo to operate, but it is much smaller than the degree of ionization in the inner nebula. The ratio  $\eta_{\text{turb}}/\eta$  is larger than 1, but not overwhelmingly so, and  $h/r$  is smaller than close to the protosun. Consequently the ratio  $\mathcal{T}_b/\mathcal{T}_v$  is typically smaller than 1 and viscous stress dominates fluctuating Maxwell stress.

In summary, the present calculation found that the viscous nebula can generate magnetic fields that would have a substantial effect on the structure and evolution of the nebula. Those effects would vary during the nebula evolution and would be most important in the inner parts of the nebula, which in the context of this

paper can extend up to 10 AU at certain evolutionary stages, but are typically smaller.

## V. SUMMARY AND DISCUSSION

The *potential* importance of magnetic fields on the structure and evolution of the solar nebula has been frequently pointed out, because magnetic torque can be, *in principle*, highly effective in removing angular momentum from a nebula. At the same time it is also widely believed that magnetic fields will probably largely diffuse out of forming nebula long before they could become dynamically important. Interestingly, those issues, even though they have been frequently raised, have not been checked out quantitatively. The purpose of this paper was to investigate, as quantitatively as presently possible, the existence and dynamical importance of nebular magnetic fields.

We have demonstrated in Section III that, indeed, the interstellar magnetic field compressed during solar nebula

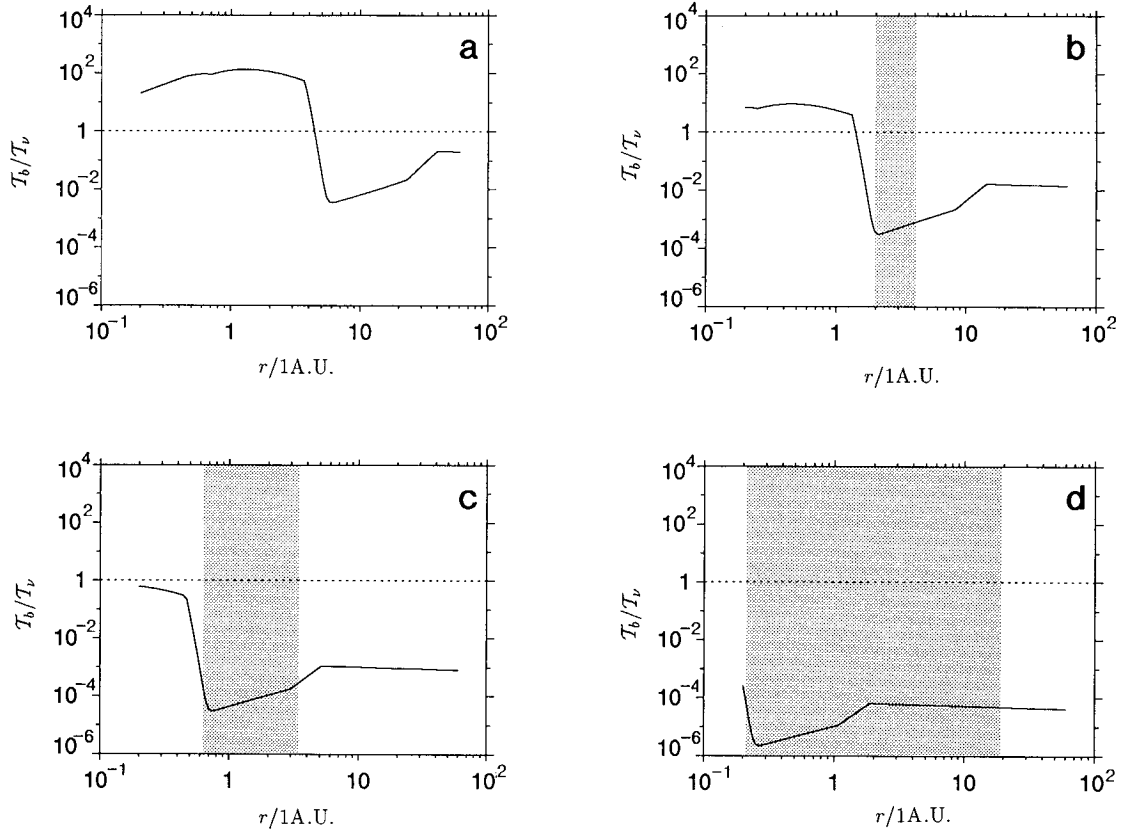


FIG. 6. The ratio of the fluctuating Maxwell stress to the viscous turbulent stress as a function of distance from the protosun. (a–d) correspond to calculations for four nebula models with  $\alpha_{ss} = 0.06$  and  $\dot{M}$  equals to 10, 1, 0.1, and 0.01, respectively. Grain size equal to 1 cm and equipartition strength large-scale magnetic field are assumed. The shaded areas represent regions of the nebula where magnetic field cannot be maintained.

formation would not survive long enough to be important. However, we have also shown that magnetic fields in the primordial nebula can be contemporaneously generated and maintained over a relatively long period of time by means of MHD dynamo. We have investigated a very broad range of nebula models and come to the conclusion that the vast majority of physically relevant models permit magnetic field regeneration in at least some parts of the nebula. There are usually two distinct regions of nebular disk where a dynamo can operate: the inner region, where the magnetic field couples to gas due to relatively high thermal ionization, and the outer region, where this coupling is achieved due to nonthermal ionization. Most models also show the existence of the intermediate region, “the magnetic gap,” where neither thermal nor nonthermal sources can produce enough ionization to provide the necessary coupling between magnetic field and the gas. The existence of such a gap is a robust feature of the nebular dynamo; at the same time the location and width of the gap change substantially from one model to another. Thus, at different evolutionary times, described by different nebula models, a magnetic field is excluded from dif-

ferent parts of the nebula. This is an important result inasmuch as it does not contradict the existence of magnetic field at the location of the present-day asteroid belt at some time during nebula evolution. Earlier, less complete calculations (Paper I) suggested that magnetic fields were absent from this region. This posed a problem because our only piece of evidence for the presence of nebular magnetic field comes from the observation that carbonaceous chondrites, relics from the nebular epoch of the Solar System, which presumably formed in the asteroid belt, have been magnetized in fields with intensities in the range 0.1 to 1 Gauss (Butler 1972, Brecher 1972). This is no longer a problem because, on the basis of our present calculations, we can envision the preasteroid region to be magnetized at a certain evolutionary stage.

Although, we were able to *calculate* the nebula location where magnetic fields can be magnified by a dynamo, we have no possibility of calculating the strength of the equilibrium field. At present we can only *estimate* the saturation strength by considering a number of different dynamical constraints on the growth of magnetic fields. The considerations in section IV show that the saturation

strength of the large-scale, magnetic field is very likely to be in the equipartition with the turbulent kinetic energy. It is interesting to observe that if we accept the equipartition value for the equilibrium strength of large-scale nebular fields, then values of about 1 Gauss at the preasteroid region of the nebula are obtained for the models that permit the existence of the field there (see Fig. 5). This is consistent with estimation given by Levy *et al.* (1991) and the field strengths inferred from magnetization of carbonaceous chondrites.

In the inner parts of the nebula, the intense magnetic fluctuations may exceed equipartition strengths on short time and length scales. Small “parcels” of nebular gas that are permeated by them would rise to the disk surface due to magnetic buoyancy where they will form “coronal” loops. Those loops would rapidly reconnect giving rise to energetic flares on the disk surface. It was postulated by Levy and Araki (1988) that chondrules melted as a result of being exposed to energetic particles from such flares. Solar-type, pre-main-sequence stars show diverse flaring activities, and it is conceivable that at least some of those flares may have originated from the disk, rather than from the central star.

In Section I we pointed out that MHD,  $\alpha\omega$ -type dynamos constitute, at present, the best framework in which to discuss the issues of existence and character of nebular magnetic fields. We also mentioned that under nebular conditions (which are definitively very different from conditions in accretion disks around compact stars) the BH instability—a rival framework to discuss issues of nebular magnetic field—is unlikely to operate. We now elaborate on this statement. There are two conditions that must be met in order for BH instability to work, both of which are discussed in the original paper by Balbus and Hawley.

First, the instability has been derived under the assumption of *perfect* conductivity; however, the solar nebula is actually a rather poor electrical conductor and BH instability will be damped by sufficiently high magnetic diffusivity. The condition that damping is unimportant is  $v_A^2 \gg 3\omega(\eta + \eta_{\text{turb}})$ . Second, there exists a critical wavelength in BH instability, below which the instability is suppressed. Clearly, this critical wavelength must be smaller than the disk thickness, leading to the condition  $v_A < \sqrt{6}C_s/\pi$  (see the Balbus and Hawley paper). These two conditions must be met *simultaneously* in order for BH instability to exist. In addition, even if those conditions are met *simultaneously*, the strength of magnetic field they bracket must be dynamically feasible. Assuming that magnetic diffusivity is dominated by turbulent dissipation, the dissipative damping condition has a simple form:  $B \gg \sqrt{3}B_{\text{eq}}$ . The critical wavelength conditions can be reduced to  $B < \sqrt{6/\pi^2\alpha_{\text{ss}}}B_{\text{eq}}$ . For most nebula models these two conditions cannot be met simultaneously. We may stretch (without any obvious physical reason) the

dissipative damping condition to  $B > \sqrt{3}B_{\text{eq}}$  instead of  $B \gg \sqrt{3}B_{\text{eq}}$ , and find that there is a very narrow range of magnetic field strengths (for  $\alpha_{\text{ss}} = 0.08$  this range is  $1.73B_{\text{eq}} < B < 2.76B_{\text{eq}}$ ) that permits both conditions to be met simultaneously. Note, however, that those strengths are higher than the equipartition value. We conclude that the regime under which BH instability may operate in the solar nebula is very restrictive, making the applicability of this instability to the nebular disk questionable.

Are generated magnetic fields strong enough to alter the structure of the nebula prescribed by turbulent viscosity? We showed in section IV that large-scale magnetic fields would not dominate viscous stress in the process of angular momentum transport. Nevertheless, for some stages in nebula evolution they may exert stress about equal to viscous stress. Moreover, the additional magnetic stress acts only in the parts of the nebula where magnetic fields exist. Those parts would experience faster evolution compared to the parts located in the magnetic gap. Altogether, the structure and the time evolution of the nebula with magnetic fields included may be very different from what the present models indicate. In the inner parts of the nebula, Maxwell stress due to small-scale, random fields can dominate viscous stress. The angular momentum transport due to the action of small-scale fields does not conceptually resemble the magnetic field line tension action we have envisioned for large-scale fields. Instead, it resembles the “magnetic viscosity” scenario envisaged by Eardley and Lightman (1975). Regardless of the particular mechanism by which small-scale fields actually transport the angular momentum, they are an important factor, and no model of the solar nebula is complete without taking them into consideration. We thus have demonstrated the need to incorporate magnetic fields into the next generation of nebular models. This is difficult inasmuch as in doing this we cannot rely exclusively on heuristic dynamical constraints, but have to actually calculate a dynamo structure to find *where* in the nebula magnetic fields can actually exist. Note that this is not an issue in accretion disks around compact stars, where it is relatively easy to demonstrate that dynamo will work *everywhere* in a disk, and one can use *only* dynamical constraints to estimate magnetic transport of angular momentum.

Finally, we must emphasize that problems of nebular magnetic field existence; its generation, its magnitude as a function of space and time, and the role it plays in nebular dynamics are incredibly complex. We can only start to address them by making a lot of assumptions and conceptual simplification. Nevertheless, the results of our calculations seem to be robust, because they reflect what we presently think are the basic physical realities of the solar nebula. There are two new concepts currently under consideration, which may somewhat modify the outcome

of our calculations. First, in the nebular region where cosmic rays are a dominant source of ionization, one should consider the interaction of cosmic rays with generated magnetic fields. Second, at certain stages of nebula evolution, some additional ionization mechanisms, such as ionization by collisions with grains falling onto the nebula, ionization by bow shocks in front of planetesimals, and ionization by sound shock waves, may become important enough to substantially change the degree of ionization of nebular gas. We will address those issues in future papers.

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